Impact of Transmit Antenna Array Geometry on Downlink Data Rates in MIMO Systems

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Abstract—For a cellular system with a fixed number of transmit antennas at each base station, we investigate how the data rate on the downlink varies as a function of the geometry of the transmit array and as a function of the linear precoding that precedes the transmit array. The maximum average data rate that can be reliably transmitted over the downlink channel (i.e. the ergodic capacity of the downlink channel) is used in this paper to compare various configurations of transmit antennas at the base stations. We show that the 3GPP Spatial Channel Model [7] can be used to semi-analytically evaluate the ergodic capacity for any configuration of transmit antennas. Based on the evaluation of ergodic capacity, we will compare several configurations of transmit antennas for a total of 2, 4, and 8 transmit antennas in a typical macro cellular environment.

I. INTRODUCTION

The use of multiple antennas at the base stations is being proposed to enhance the performance of cellular wireless systems in several standardization bodies (e.g. 3GPP UMTS [2], 3GPP Long Term Evolution of UMTS [1], WiMax 802.16 [3]). Over the last decade, extensive research in the physical-layer aspects of multi-antenna systems has been carried out [4]–[6]. This type of research has traditionally focused on finding transmitter and receivers that enable the transmission of the highest data rate over a given MIMO channel for a fixed total transmitted power.

The maximum average data rate that can be reliably transmitted over a MIMO channel (i.e. the ergodic capacity [12] of the MIMO channel) is a function of the statistics of the MIMO channel. The statistics of the MIMO channel in turn are affected by the configuration of the antennas at the transmitter, the configuration of antennas at the receiver, and by the statistical characteristic of the medium surrounding the transmitter and the receiver. Rather than taking the statistics of the channel as given, in this paper we focus on how various configurations of transmit antennas at the base station (BTS) affect the statistics of the downlink MIMO channel and how this in turn will affect the ergodic capacity on the downlink channel.

For a given number of transmit antennas, a particular configuration of these antennas is specified by the location of the transmit antennas (i.e. the geometry of the transmit antenna array) and by any precoding that might precede the transmit antennas [8], [9] (the exact nature of this precoding will be explained further in Section II).

To determine the effect of the transmit antenna configuration on the performance of the downlink, we firstly present a mathematical parametrization of a large class of transmit antenna configurations, secondly we will express our goodness criteria (the ergodic capacity) as a function of these parameters, and finally we will compare the various choices for these parameters based on their resulting ergodic capacity.

Multiple transmit antennas at BTS can be used to enhance the downlink performance in different ways. For example, multiple transmit antennas can be used to increase the data rate received by the users with the highest SNR (i.e. improve the peak rate). Alternatively, multiple transmit antennas could be used to increase the data received by the users with the lowest SNR (i.e. improve the coverage). Our results will show that some transmit antenna configurations are better suited for improving the peak rate, some transmit antenna configurations are better for improving the coverage, and other transmit antenna configurations achieve a tradeoff between improving the peak rate or improving the coverage.

The rest of this paper is organized as follows. In Section II, we present our model for the downlink MIMO channel, and we present a parametrization for a large class of transmit antenna configurations. In Section III, our criteria for comparing various configurations of transmit antennas (i.e. ergodic capacity of the MIMO channel) is also presented. For 2, 4, and 8 transmit antennas at the BTS, the ergodic capacity for various configurations of these transmit antennas is computed in Section III. Finally in Section IV, we draw some conclusions regarding the impact of various transmit antenna configurations in terms of improving the peak data rate and improving the coverage on the downlink of cellular systems.

II. SYSTEM MODEL

Our system model for the downlink is depicted in Fig. 1. The mobile is equipped with \(N_{rx}\) receive antennas, and the \(k\)-th receive antenna is located at spatial coordinates \(\mathbf{R}_k\). The BTS is equipped with \(N_{tx}\) transmit antennas, and the \(i\)-th transmit antenna is located at spatial coordinates \(\mathbf{r}_i\). In the frequency domain, the output of the \(k\)-th receive antenna is denoted by \(Y_k(f)\), and the signal transmitted from the \(i\)-th transmit antenna is denoted by \(X_i(f)\); hence, \(\mathbf{X}(f) = [X_1(f) \ldots X_{N_{tx}}(f)]^T\) and \(\mathbf{Y}(f) = [Y_1(f) \ldots Y_{N_{rx}}(f)]^T\) are the vector signal transmitted from the BTS antennas, and the vector signal receive at the mobile antennas respectively. We can relate \(\mathbf{Y}(f)\) to \(\mathbf{X}(f)\) according to:

\[
\mathbf{Y}(f) = \mathbf{G}(f) \mathbf{X}(f) + \mathbf{V}(f),
\]  
(1)
where $\mathbf{V}(f)$ is vector of size $(N_{tx} \times 1)$ whose $k$-th element is the additive white Gaussian noise (with variance $N_0$) at the $k$-th receive antenna, and $\mathbf{G}(f)$ is the transfer function matrix ($\text{size } (N_{tx} \times N_{tx})$) of the medium between the transmit antennas and the receive antennas. The $(i, k)$ element of matrix $\mathbf{G}$ is denoted by $G_{i,k}(f)$ which makes it clear that the $(i, k)$ element of $\mathbf{G}$ is the transfer function of the single-input/single-output channel between the $i$-th transmit antenna located at $\mathbf{r}_i$ and the $k$-th receive antenna located at $\mathbf{r}_k$. It is then obvious that $\mathbf{G}$ depends on the geometry of the transmit antenna array and the geometry of the receive antenna array.

We will assume that the transmit antennas are preceded by a linear precoding matrix $\mathbf{W}$ of size $(N_{tx} \times M)$, and that the transmitter is not aware of the specific realization of $\mathbf{G}$; however, we assume that the transmitter is aware of the statistics of $\mathbf{G}$. This implies that $\mathbf{W}$ is a function of the statistics of $\mathbf{G}$, and $\mathbf{W}$ is not a function of the specific realization of $\mathbf{G}$.

The input to the precoder is an $(M \times 1)$ vector $\mathbf{S}(f) = [S_1(f) ... S_M(f)]^T$ with independent, identically distributed entries. Note that $S_m(f)$ is the symbol that is transmitted from the $m$-th stream. The covariance matrix of $\mathbf{S}(f)$ is then given by:

$$K_{ss}(f) = E\{\mathbf{S}(f)\mathbf{S}(f)^H\} = (P_0/M)I_{M,M},$$

where $I_{M,M}$ denotes an identity matrix of size $(M \times M)$, and $P_0$ is the total power in $\mathbf{S}(f)$. The vector signal that is finally transmitted from the transmit antennas is $\mathbf{X}(f)$, and the covariance matrix of $\mathbf{X}(f)$ is:

$$K_{xx}(f) = (P_0/M)\mathbf{WW}^H,$$

where to make the total transmitted power equal to $P_0$, we will require that $||\mathbf{W}||^2 = M$.

For a given transmit antenna geometry (i.e. a given $\{\mathbf{r}_i\}_{i=1}^{N_{tx}}$) and a given receive antenna geometry (i.e. a given $\{\mathbf{r}_k\}_{k=1}^{N_{rx}}$), the statistics of $\mathbf{G}$ is determined by the characteristic of the environment surrounding the BTS and the mobile. We will use the 3GPP Spatial Channel Model [7] (3GPP SCM) for macro cellular environments to obtain the statistics of $\mathbf{G}$ as a function of the transmit antenna geometry and the receive antenna geometry. More specifically, for a given transmit antenna geometry, and receive antenna geometry, the 3GP SCM can be used to generate many realization of the random matrix $\mathbf{G}$ where the statistics of this random variable is determined by the given geometries of the transmit and receive antennas.

In the remainder of paper, we will assume that the carrier frequency is $2$ [GHz], and we assume that the total bandwidth of the channel is approximately $20$ [MHz].

The maximum average data rate that can be reliably transmitted over $\mathbf{G}$ is referred to as the [12] ergodic capacity of $\mathbf{G}$, and this ergodic capacity can be expressed as:

$$C = E_G\left\{\frac{1}{2} \int \log \det \mathbf{I} + \frac{1}{N_0} \mathbf{G}(f)\mathbf{K}_{xx}(f)\mathbf{G}^H(f) | df \right\}$$

$$= E_G\left\{\frac{1}{2} \int \log \det \mathbf{I} + \frac{P_0}{M N_0} \mathbf{G}(f)\mathbf{W}\mathbf{W}^H\mathbf{G}^H(f) | df \right\},$$

where $E_G(.)$ is the expectation operator with respect to the statistics of $\mathbf{G}$, and to get to Eq. (4) we have used Eq. (3).

We would like to emphasize several important properties of ergodic capacity that justify why we use it as a criteria to compare different transmit antennas configurations. Firstly, it is well-known [12] that no transmitter/receiver can transmit data over the MIMO channel at a higher rate than the ergodic capacity $C$ in Eq. (4). Secondly, over the last decade practical transmitters and receivers have been developed that are capable of reliably transmitting data over MIMO channels at rates very close to the ergodic capacity of these MIMO channels [4], [5]. Based on the availability of such capacity-achieving transmitters/receivers, it is natural to try to find configurations of transmit antennas and of receive antennas that maximize the ergodic capacity (for a given number of transmit and receive antennas).

From Eq. (4), we see that $C$ depends on the ratio $P_0/N_0$; hence, it is useful to define $SNR = P_0/N_0$.

For a given number of transmit antennas and given number of receive antennas, we see from Eq. (4) that for each value of $SNR$, the ergodic capacity $C$ is a function of the following triplet $\{\mathbf{W}; \{\mathbf{r}_i\}_{i=1}^{N_{tx}}; \{\mathbf{r}_k\}_{k=1}^{N_{rx}}\}$. For a given number of transmit antennas, a given number of receive antennas, and a given $SNR$, one could in principal find the triplet $\{\mathbf{W}; \{\mathbf{r}_i\}_{i=1}^{N_{tx}}; \{\mathbf{r}_k\}_{k=1}^{N_{rx}}\}$ that maximizes the ergodic capacity expression of Eq. (4); however, this optimization is rather intractable. To gain some understanding of the behavior of $C$ as a function of the transmit antenna configuration, we will next compute $C$ for a handful of choices for $\{\mathbf{W}; \{\mathbf{r}_i\}_{i=1}^{N_{tx}}\}$.

The motivation for these choices will become clear in the next section.

To be precise, the pair $\mathbf{W}$ and $\{\mathbf{r}_i\}_{i=1}^{N_{tx}}$ jointly define a transmit antenna configuration. In the rest of this paper, a given transmit antenna configuration refers (by definition) to a particular choice for $\{\mathbf{W}; \{\mathbf{r}_i\}_{i=1}^{N_{tx}}\}$.
III. ERGODIC CAPACITY OF SEVERAL TRANSMIT ANTENNA CONFIGURATIONS

To isolate the impact of different transmit antenna configurations on $C$, we will next fix the receive antenna geometry. More specifically, in the remainder of this paper we shall assume that the mobile is always equipped with two receive antennas, and we shall assume that the distance between these receive antennas is $\frac{\lambda}{2}$, where $\lambda$ is the wavelength at the carrier frequency. This choice for the geometry of the receive antennas is partially motivated by the practical constraints on the size and cost of mobiles.

Each of the following three sub-sections focus on the behavior of $C$ for a fixed number of transmit antennas. In each of these sub-sections, the value of $C$ is computed for various configurations of the available transmit antennas (with the total number of transmit antennas kept fixed). For each transmit antenna configuration, the value of $C$ is computed over a large range of $SNR$ values.

The computation of $C$ in Eq. (4) is carried out numerically as follows. Referring to Eq. (4), one can view $C$ as the expected value of a functional of $G$. This suggests generating a large number of realizations of $G$ (let $G_i(f)$ denote the $i$-th such realization) using the 3GPP SCM. When generating these realizations of $G(f)$, the value of $(W; \{r_i\}_{i=1}^N)$ is fed to the 3GPP SCM to generate the correct statistics for these realizations. Next, we compute the following functional for each generated $G_i(f)$:

$$z[l] = \frac{1}{2} \int \log det[I + \frac{P_0}{MN_0} G_i(f) WW^\dagger G_i(f)] df.$$ (5)

The final value of $C$ is simply obtained as the empirical mean of $z[l]$, i.e. $C = \frac{1}{L} \sum_{l=1}^L z[l]$, where $L$ is the total number of realizations.

A. Two Transmit Antennas

With just two transmit antennas, the geometry of the transmit antenna only depends on the distance between these two antennas (i.e. $d = ||r_1 - r_2||$). As two extreme choices of $d$, we will consider $d = \frac{\lambda}{2}$ and $d = 10\lambda$.

For notational simplicity, let us define the following $[Q, 1]$ vector:

$$\Phi_Q = \left[1, e^{j2\pi\alpha\Delta_1}, e^{j2\pi\alpha\Delta_2}, \ldots, e^{j2\pi\alpha\Delta_Q}\right]^T,$$ (6)

where $\Delta_i = r_i - r_1$, and $\alpha$ is a row, unit vector pointing from the center of the transmit antenna array at the BTS to the center of the receive antenna array at the mobile.

With $d = 10\lambda$, the two transmit antennas are approximately independent; hence, a natural choice for $W$ is $W = I_{2,2}$. The motivation for this choice is as follows. Firstly, note that $W$ can be viewed as shaping the covariance matrix of the transmitted signal $K_{xx}(f)$ through Eq. (3); hence, one should choose $W$ to match $K_{xx}$ to the statistics of the channel $G$. When the transmit antennas are placed far apart, the entries of $G$ becomes approximately mutually independent; hence, $G$ does not possess any particular statistical structure in this case. Without any particular structure in $G$, there is no reason to impose a particular structure on the transmitted signal; hence, the transmitted signal vector in this case should have independent entries, and this independence can be achieved by setting $W$ equal to an identity matrix. The above choice for $W$ is consistent with the findings in [13], [14].

With $W = I_{2,2}$, there will be two columns in $W$; two independent streams will be transmitted to the mobile, and there will be no array gain. The ergodic capacity for this transmit antenna configuration is depicted in Fig. 3 in blue with circles.

With $d = \frac{\lambda}{2}$, the two transmit antennas can be used to form a beamformer pointing in the direction of the mobile if we set the precoding matrix $W$ equal to a $(2, 1)$ vector of phases. More specifically, we set $W = \Phi_2$. In this case, $W$ has just one column; hence, $M = 1$ (see Fig. 1) and only one stream is transmitted to the mobile. In an environment with moderate angular spread (e.g. a typical macro cellular environments in the 3GPP SCM), the mobile will see an array gain of approximately $3$[dB] with this 2-element beamformer. The above choice for $W$ is consistent with the findings in [14]; since, with closely-spaced transmit antennas, the dominant eigen vector of $E[G(f)G(f)^\dagger]$ is approximately equal to $\Phi_2$.

The ergodic capacity for this transmit antenna configuration is depicted in Fig. 3 in solid red.

Comparing the ergodic capacity of the two transmit antenna configurations in Fig. 3, we see that the beamforming configuration (i.e. $d = \lambda/2$) results in the highest data rate for users with low $SNR$ (i.e. those with $SNR$ below $5$[dB]), on the other hand, the configuration with the two antennas placed far apart (i.e. $d = 10\lambda$) results in the highest data rate for users with high $SNR$ (i.e. those with $SNR$ above $5$ [dB]).

Users with low $SNR$ are power-limited, and they benefit greatly from a boost to their received power (as is the case with the beamforming configuration above), and these users do not benefit much from having more than one independent stream transmitted to them (as is the case with the widely-spaced configuration above). Conversely, users with high $SNR$ are degrees-of-freedom limited; hence, these users benefit greatly from having more than one stream transmitted to them (as is the case with the widely-spaced configuration above), and these users do not benefit much from a boost to their received power (as is the case with beamforming configuration above).

From the discussion above, we conclude that placing all the antennas close together with just one transmitted streams (i.e. with precoding matrix corresponding to one angular beamformer) tends to mostly benefit the users with the lowest $SNR$, and placing the antennas far apart with several independent transmitted streams tends to mostly benefit the users with the highest $SNR$.

When the number of transmit antennas is more than two, a few other configurations of transmit antennas can be constructed. Assume that the number of transmit antennas can be expressed as a product of two integers:

$$N_{tx} = M \times Q.$$ (7)
where $M > 1$ and $Q > 1$. In this case, one can group the $N_{tx}$ transmit antennas in $M$ clusters where each cluster has $Q$ antennas. The $Q$ antennas in each cluster can be configured to form a linear array with $\frac{\lambda}{2}$ spacing between consecutive antenna elements within each cluster. Different clusters could in turn be placed far apart such that the intra-cluster spacing is $10\lambda$. In this case, $\mathbf{W}$ becomes a block diagonal matrix (with $M$ blocks on its diagonal) of size $(N_{tx}, M)$, i.e.

$$W = \text{diag}(\Phi_Q; \ldots; \Phi_Q).$$

Fig. 2 schematically illustrates one such configuration with $N_{tx} = 4$, $M = 2$ (number of clusters), and $Q = 2$ (number of antennas per cluster).

### B. Four Transmit Antennas

With four transmit antennas, we consider three transmit antenna configurations. In the first configuration, the four transmit antennas are placed on a line with $10\lambda$ spacing between consecutive antennas, and with $\mathbf{W} = I_{4,4}$ (i.e. 4 independent streams are transmitted). In the second configuration, the four transmit antennas are used to form a linear array with $\frac{\lambda}{2}$ spacing between the consecutive antenna elements, and $\mathbf{W} = \Phi_4$. In the third configuration, the antennas are divided into two clusters of beamformers (see Fig. 2), and $\mathbf{W} = \text{diag}(\Phi_2; \Phi_2)$. The ergodic capacities for these three configurations with $N_{tx} = 4$ are plotted in Fig. 4.

From Fig. 4, we see that the configuration with two beamformers (each beamformer having two antennas) results in the highest data rate for user with $SNR$ more than 2.5[dB], and the configuration with just one beamformer results in the highest data rate for users with $SNR$ less than 2.5[dB]. It is interesting to note there is no $SNR$ for which the configuration with 4 widely-spaced antennas results in the highest data rate. Furthermore, we see that for $SNRs$ below 2.5[dB], using the configuration with 2 beamformers results in approximately 1[dB] loss compared to using the configuration with one beamformer.

### C. Eight Transmit Antennas

With eight transmit antennas, we consider four transmit antenna configurations. In the first configuration, all eight transmit antennas are placed on a line with $10\lambda$ spacing between consecutive antennas, and with $\mathbf{W} = I_{8,8}$ (i.e. 8 independent streams are transmitted). In the second configuration, the eight transmit antennas are used to form a linear array with $\frac{\lambda}{2}$ spacing between the consecutive antenna elements, and $\mathbf{W} = \Phi_8$. In the third configuration, the antennas are divided into two clusters of beamformers (i.e. $M=2$ and $Q = 4$ in Eq. (8), and $\mathbf{W} = \text{diag}(\Phi_4; \Phi_4)$. In the fourth configuration, the antennas are divided in four clusters of beamformers (i.e. $M = 4$ and $Q = 2$ in Eq. (8), and $\mathbf{W} = \text{diag}(\Phi_2; \Phi_2; \Phi_2; \Phi_2)$). The ergodic capacities for these four configurations with $N_{tx} = 8$ are plotted in Fig. 5.

From Fig. 5, we see that the configuration with two beamformers results in the highest data rate for users with $SNR$ more than -2.5[dB], and the configuration with one beamformer results in the highest data rate for users with $SNR$ less than -2.5[dB]. Interestingly, we see that for $SNRs$ less than -2.5[dB], using the configuration with 2 beamformers results in approximately 0.5[dB] loss compared to using the configuration with one beamformer.

In a practical system, the configuration of the transmit antennas at the base stations must be fixed. Secondly, in a cellular system, there is a wide distribution for the $SNR$ of the users. With frequency reuse 1, the users at the edge of the cell can experience $SNR$ as low as -10[dB], and the users close to their serving cell can experience $SNR$ up to 20[dB]. For a given number of transmit antennas, ideally we would like to find one configuration of these transmit antennas that results in the highest data rate (among all possible transmit antenna configurations with the same the given number of transmit antennas) for all the users (i.e. for all $SNRs$).

The results in this section suggest that with a small number (e.g. 2) of transmit antennas at the base stations, there is no one configuration that results in the highest data rate for all $SNRs$. With a few transmit antennas, the configuration with just one beamformer results in significantly higher data rate for user with low $SNR$. Unfortunately, this configuration will also result in the lowest data rate for the users with high $SNRs$. In other words, with a small number of transmit antennas, there is a hard tradeoff between peak data rate and coverage.

With a large number of transmit antennas (e.g. 8) at the base stations, the picture is more favorable. From Fig. (5), we see that the configuration with 2 beamformers (each beamformer having 4 antenna elements) results in the highest data rate for almost all users. At very low $SNRs$ (i.e. $SNR$ below -2.5[dB]), there a is very slight (0.5[dB]) loss in $SNR$ from using this configuration with 2 beamformers. In other words, the transmit antenna configuration that groups the transmit antennas in two beamformers and places these two beamformers 10 $\lambda$ apart simultaneously results in the highest data rate for the users with high $SNRs$ (i.e. improved peak rate) and the highest data rate for user with low $SNR$ (i.e. improved coverage).

### IV. Conclusion

Using ergodic capacity as a measure, we compared various configurations of transmit antennas at cellular base stations. For any number of transmit antennas, we offered a parametrization of a large class of transmit antenna configurations in terms of a precoding matrix $\mathbf{W}$ and the location of the transmit antennas. We showed that the 3GPP Spatial Channel Model can be used to semi-analytically evaluate the ergodic capacity for any combination of the above parameters (i.e. for any configuration of transmit antennas). Based on their resulting ergodic capacity, we compared various configurations of transmit antennas for a total of 2, 4, and 8 transmit antennas in a typical macro cellular environment. For the case with 8 transmit antennas at the base stations, we found that one transmit antenna configuration (see Fig. (5) ) results in the highest data rate for almost all users in the cellular system.
REFERENCES


Fig. 2. One configuration with 4 transmit antennas

Fig. 3. Capacity of different configurations with 2 TX antennas

Fig. 4. Capacity of different configurations with 4 TX antennas

Fig. 5. Capacity of different configurations with 8 TX antennas